Econometrics 2b: Handout #5

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1 A General ARMA Process

- We can now formulate a more general autoregressive moving average process, ARMA(p, q)

$$Y_t = \mu + \theta_1 Y_{t-1} + ...\theta_p Y_{t-p} + \varepsilon_t + \alpha_1 \varepsilon_{t-1} + ... + \alpha_q \varepsilon_{t-q}$$

$$\varepsilon_t \sim IID(0, \sigma^2).$$

- For nonstationary series, we formulate an ARIMA(p, d, q) model.
- For a nonstationary, seasonal time series, we formulate an ARIMA$(p, d, q) \times SAR(s) \times SMA(s)$ model.

2 Estimating ARIMA Models

The goal of ARIMA analysis is a parsimonious representation of the process governing the residual. You should use only enough AR and MA terms to fit the properties of the residuals correctly.

2.1 The Box-Jenkins Methodology

This methodology can be applied to stationary series only. So, you must first deal with unit roots and stochastic seasons.

1. Identification
   (a) Address seasonality, s

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(b) Determine order of integration, d

(c) Find appropriate values of p and q

2. Estimation

(a) Pure $AR(p)$ models can be estimated (consistently) using OLS, non-linear OLS or maximum likelihood.

(b) Pure $MA(q)$ models can be estimated (consistently) using non-linear OLS or maximum likelihood.

(c) ARIMA models, $AR(p)$ models and $MA(q)$ models can all be estimated (consistently) using non-linear OLS and maximum likelihood.

i. The **arima** function in $\mathbb{R}$ uses maximum likelihood (see **?arima** for more info).

note: Consistency is an asymptotic property (i.e., it hold for large samples). This does not guarantee unbiasedness in small samples when estimating ARIMA models! We will discuss this topic in more detail later.

1. Diagnostics

(a) model specification

(b) autocorrelations

(c) ARCH

(d) normality