Mathematical model and solution method of ordering controlled virtual assembly plants and service centres

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Abstract

In an earlier article [1] we have presented a new production model with the related location model under circumstances created by the global economic crisis. In this paper we should like to focus on two mathematical models that reveal the internal structure of this location connected production in details, the model of group formation, the definition of the notion of best clustering as well as the selection of a possible center together with the related assignment model. In this model we attempted to simplify the objective functions of [1] and [5] with the minimal distortion. From the point of view of defining the clusters this simplified objective function can be well applied without significant differences would appear. The task related to the model offers more solution methods [4]; however, we do not examine this scope of question as this will be the subject of another study.

Key words: clustering, mathematical model, natural groups, pure remote manufacturing

JEL Classification: C61 - Optimization Techniques; Programming Models

1 Introduction

Pure Remote Manufacturing.

Let be given *n* demand places and m possible places. Let us define the demand places belonging together and assign the possible manufacturing facilities to the natural demand place groups to be formed. All this should be made in an "optimal" way.

In the solution we need to search for production places close to the demand places which are able to satisfy the full demand. In this case, if needed, only the strategic parts should be got to the given assembly plant and the question of supplier should be out of interest as it would belong to the responsibility of the local plant. Apparently, the quality control should be achieved by the centre. Outbound transportation would be accomplished also by local shipping companies.

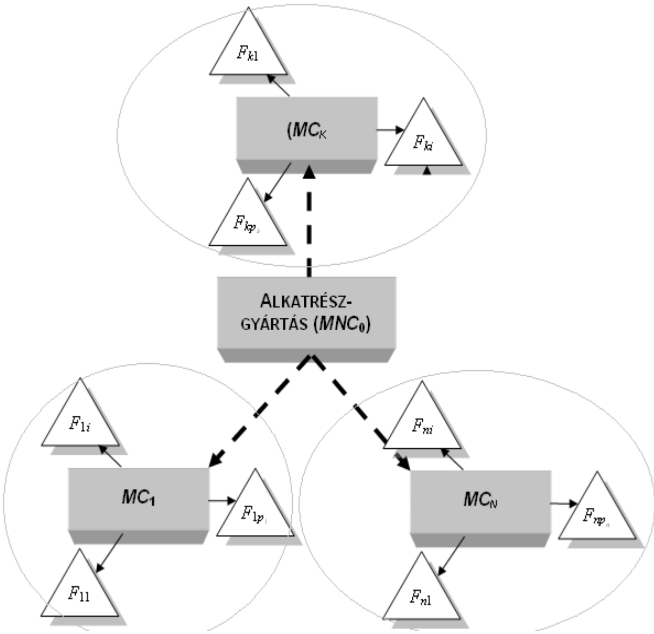


Fig. 1 Pure remote manufacturing

2 Mathematical model

2.1 Formation of "natural groups"

The mathematical model related to the formation of the group.
Let us suppose we plan to form *c* demand groups by taking *n* demand places into account ($c \le n$).
Let us have

$$\mathrm{T} = \lfloor \mathrm{t}_{\mathrm{ij}} \rfloor \tag{1}$$

to denote the length of path (edge) in relation with $i \to j$. More detailed refer to [2, 3].

Definition 1
Given

$$\mathbf{D} = \lfloor d_{ij} \rfloor_{n \times n} \tag{2}$$

$$d_{ij} = k_{ij} \cdot t_{ij}, (i = 1, \ldots n; j = 1, \ldots, n)$$

as the capacity of $i \to j$ edge while k_{ij} should mean the specific transportation cost related to one distance unit in the $i \to j$ relation. This cost factor should be a value defined upon practical experiences that is dependent on the quantity (volume) to be transported, on the characteristics of vehicle(s) as well as on the conditions of transportation. For its detailed definition refer to [1].

Remarks
1. In this case the conditions related to the generalized distance have been satisfied as

$$k_{ij} \geq 0. \tag{3}$$

2. In this case the similarity indicator will appear as (i.e. two points will be in the same cluster if the similarity is better)

$$D_{ij} = \frac{1}{d_{ij}}, \tag{4}$$

D_{ij} is the capacity of the reverse distance between i and j.

Let be the value of

$$\mathbf{X} = \lfloor x_{ij} \rfloor \ (i = 1, \dots n;\ j = 1, \dots, n) \tag{5}$$

- 1, if the edge ij is used
- 0, if not

in the $i \to j$ relation.

Denote

$$\mathbf{y} = [y_1, \dots, y_n]^* \tag{6}$$

where y_i is the number of cluster.

In this case

$$0 \leq y_i \leq c - 1 \tag{7}$$

and if

$$x_{ij} = 1 \text{ then } y_i \neq y_j \tag{8}$$

2.2 Clustering

Arrange the edges in the order of their size related to their capacity. Assume the order will be as shown below:

$$d_{i_1 j_1} \geq d_{i_2 j_2} \geq \dots \geq d_{i_{\frac{n(n-1)}{2}} j_{\frac{n(n-1)}{2}}}, \tag{9}$$

$$y_i \in \{0, \dots, c-1\} \ (i = 1, \dots n), \tag{10}$$

$$x_{ij} \in \{0;1\} \ (i = 1, \dots n;\ j = i+1, \dots n) \tag{11}$$

$$d_{i_k j_k} x_{i_k j_k} - d_{i_{k+1} j_{k+1}} x_{i_{k+1} j_{k+1}} \geq 0 \ (k = 1, \dots, n-1) \tag{12}$$

$$(y_i - y_j)^2 - x_{ij} \geq 0 \ (i = 1, \dots n;\ j = i+1, \dots n) \tag{13}$$

$$\hat{f}(\mathbf{X}) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} d_{ij} \cdot x_{ij} \to \max. \tag{14}$$

Remarks to the model

1. (12) guarantees that a shorter edge will only be considered if all longer ones have taken into consideration:

$$d_{i_{k+1} j_{k+1}} < d_{i_k j_k}, \tag{15}$$

In this case the following may not occur:

$$x_{i_k j_k} = 0, \tag{16}$$

$$x_{i_{k+1} j_{k+1}} = 1. \tag{17}$$

2. (11) assures that in case an edge is existing between two demand points, these two point can not be in the same group. (If there is no edge existing between them they can be either in the same or also in another group; this is why not equality has been given.)
Solving the task of the model we obtain the clustering belonging to the number *c*. As a next step the clustering and the value of the objective function need to be defined for all possible *c*.
Following this, we need to find the natural clustering. Its definition is as follows:

Definition 2
Let M_c to denote clustering belonging to the number *c*. Given

$$d_c = d_{i_l j_l} - d_{i_{l+1} j_{l+1}}, \tag{18}$$

for which

$$x_{i_l j_l} = 1 \text{ and } x_{i_{l+1} j_{l+1}} = 0,$$

are satisfied for *l* and this belongs to the M_c clustering. (If $l = \frac{n(n-1)}{2}$, then $d_c = 0$).

Given

$$d_{c_0} = \max_{c=1,\dots,n-1} \left(\left| d_c - d_{c+1} \right| \right) \; (c = 1, \dots, n-1), \tag{19}$$

in which case the M_{c_0} belonging to d_{c_0} is called as natural clustering. In case there are more of this kind the one is selected for which d_{c_0} is the greatest. If also these are more than the one with the least group number is selected.

2.3 Selection of the clusters' centre

In the second step, based upon the natural clustering we may set up a center search model.
First we give the center search function. One of possible definitions of this function could be as follows.
Let C_j denote the *j*-th group belonging to the M_{c_0} clustering.
Given

$$f_{ij} = \sum_{l \in C_j} (d_{il} + p_i) \cdot q_l \tag{20}$$

where q_l is the quantity demand of the given demand place while p_i stand for the unit price of product produced at *i*-th place. The d_{il} is the matrix element of edge capacity between the demand places and possible places.

Definition 3
Let be $z_{ij} = 1$ if we transport from *i*-th possible place to the *j*-th cluster and 0 if not. Then the **Z** matrix will take up the form as shown

$$\mathbf{Z} = \lfloor z_{ij} \rfloor_{m \times n}. \tag{21}$$

Given

$$f(\mathbf{Z}) = \sum_{i=1}^{m} \sum_{j=1}^{c_0} f_{ij} \cdot z_{ij}. \tag{22}$$

which is considered to be center search function.

The center search model will take up the following form:

$$z_{ij} \in \{0;1\} \ (i = 1, \ldots, m), \left(j = 1, \ldots, c_0\right) \tag{23}$$

$$\sum_{j=1}^{c_0} z_{ij} \geq 1 \tag{24}$$

$$\sum_{i=1}^{m} z_{ij} \leq 1 \tag{25}$$

$$f(\mathbf{Z}) = \sum_{i=1}^{m} \sum_{j=1}^{c_0} f_{ij} \cdot z_{ij} \rightarrow \min . \tag{26}$$

With the help of the task belonging to the model at least one possible place will be assigned to each cluster from which their demand could be supplied.

3 Sample Task

The following simple task illustrates how easily the task can be solved in Excel.
Let us consider 4 demand places and 5 possible ones. The **D** edge capacity matrix belonging to individual demand places may pick up the following form:

$$\mathbf{D} = \begin{bmatrix} 0 & 4 & 10 & 5 \\ 4 & 0 & 2 & 7 \\ 10 & 2 & 0 & 1 \\ 5 & 7 & 1 & 0 \end{bmatrix}$$

3.1 The first model

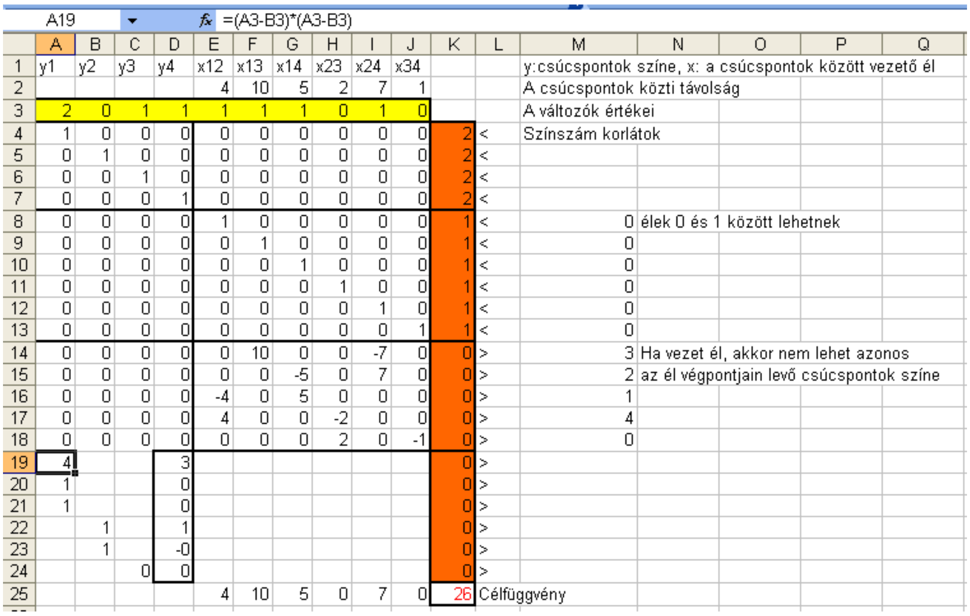
$$y_i \in \{0, \ldots, c-1\} \left(i = 1, \ldots 4\right),$$
$$x_{ij} \in \{0;1\} \left(i = 1, \ldots 4;\ j = i+1, \ldots 4\right)$$

$$\begin{array}{cccccccccccc}
 & & & & 10x_{13} & & & -7x_{24} & & \geq & 0 \\
 & & & & & -5x_{14} & & +7x_{24} & & \geq & 0 \\
 & & & 4x_{12} & & +5x_{14} & & & & \geq & 0 \\
 & & & -4x_{12} & & & -2x_{23} & & & \geq & 0 \\
 & & & & & & 2x_{23} & & -x_{34} & \geq & 0 \\
(y_1 - y_2)^2 & & & -x_{12} & & & & & & \geq & 0 \\
(y_1 - y_3)^2 & & & & -x_{13} & & & & & \geq & 0 \\
(y_1 - y_4)^2 & & & & & -x_{14} & & & & \geq & 0 \\
 & (y_2 - y_3)^2 & & & & & -x_{23} & & & \geq & 0 \\
 & (y_2 - y_4)^2 & & & & & & -x_{24} & & \geq & 0 \\
 & & (y_3 - y_4)^2 & & & & & & -x_{34} & \geq & 0 \\
 & & & & 10x_{13} & +5x_{14} & +2x_{23} & +7x_{24} & +x_{34} & \rightarrow & \max
\end{array}$$

The solution for c = 0, …,3 will result in:

Number of clusters	Clusters	d_{c_0}	d_{c_0}
1	[1;2;3;4]	10 – 7 = 3	3 – 2 = 1
2	[1],[2;3;4]	7 – 5 = 2	3 – 2 = 1
3	**[1],[2],[3;4]**	**5 – 2 = 3**	**3 – 1 = 2 !!!**
4	[1],[2],[3],[4]	2 – 1 = 1	1 – 0 = 1

Based upon the previous reasoning the cluster No. 3 will be considered as natural clustering. One of its possible Excel model is depicted in Fig. 2.


A19 =(A3-B3)*(A3-B3)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	y1	y2	y3	y4	x12	x13	x14	x23	x24	x34			y:csúcspontok színe, x: a csúcspontok között vezető él		
2					4	10	5	2	7	1			A csúcspontok közti távolság		
3	2	0	1	1	1	1	1	0	1	0			A változók értékei		
4	1	0	0	0	0	0	0	0	0	0	2	<	Színszám korlátok		
5	0	1	0	0	0	0	0	0	0	0	2	<			
6	0	0	1	0	0	0	0	0	0	0	2	<			
7	0	0	0	1	0	0	0	0	0	0	2	<			
8	0	0	0	0	1	0	0	0	0	0	1	<		0	élek 0 és 1 között lehetnek
9	0	0	0	0	0	1	0	0	0	0	1	<		0	
10	0	0	0	0	0	0	1	0	0	0	1	<		0	
11	0	0	0	0	0	0	0	1	0	0	1	<		0	
12	0	0	0	0	0	0	0	0	1	0	1	<		0	
13	0	0	0	0	0	0	0	0	0	1	1	<		0	
14	0	0	0	0	0	10	0	0	-7	0	0	>		3	Ha vezet él, akkor nem lehet azonos
15	0	0	0	0	0	0	-5	0	7	0	0	>		2	az él végpontjain levő csúcspontok színe
16	0	0	0	0	-4	0	5	0	0	0	0	>		1	
17	0	0	0	0	4	0	0	-2	0	0	0	>		4	
18	0	0	0	0	0	0	0	2	0	-1	0	>		0	
19	4			3							0	>			
20	1			0							0	>			
21	1			0							0	>			
22		1		1							0	>			
23		1		-0							0	>			
24			0	0							0	>			
25					4	10	5	0	7	0	26	Célfüggvény			

Fig. 2: A worksheet with initial data

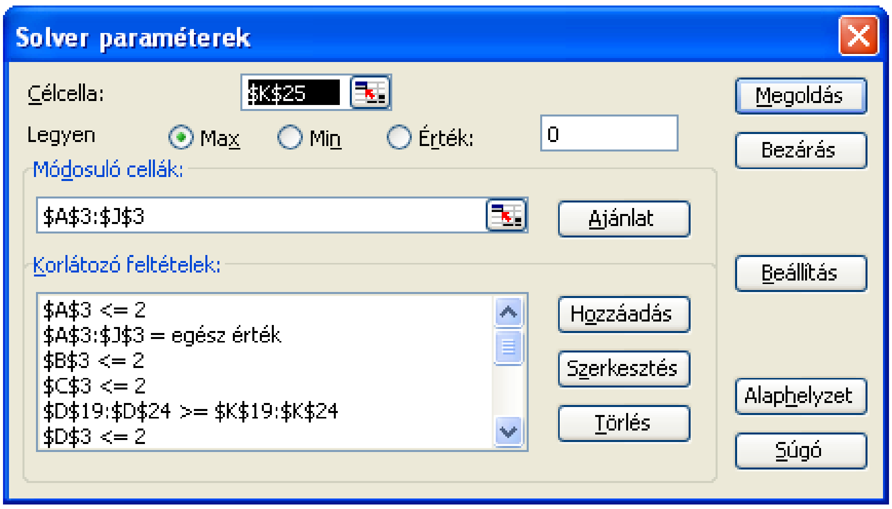


Fig. 3 Setup 1 of Solver

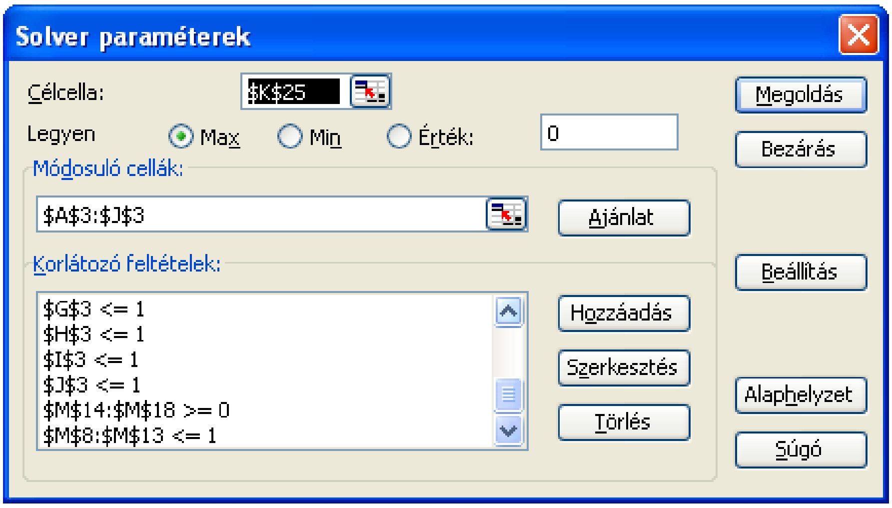


Fig. 4. Setup 2 of Solver

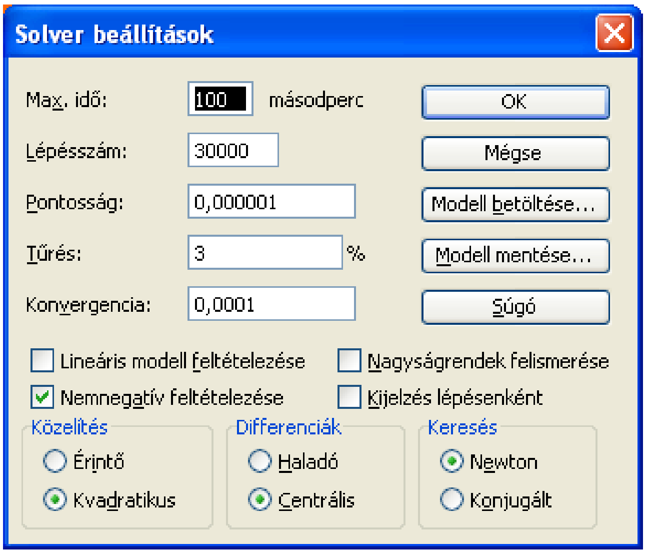


Fig. 5. Setup 3 of Solver

3.2 Second model

Solution of the task appertaining to the second model

Let the edge capacity matrix according to the model 1 be as shown below:

$$(\mathbf{D}')^{*} = \begin{bmatrix} 1 & 2 & 9 & 7 \\ 3 & 1 & 4 & 2 \\ 5 & 2 & 7 & 3 \\ 2 & 9 & 10 & 7 \end{bmatrix}$$

Let the total demand of all demand places be

$$\mathbf{q}^* = [50; \quad 80; \quad 60; \quad 70]$$

Purchase price of a product from one possible place:

$$\mathbf{p} = [100; \quad 150; \quad 200; \quad 90]^*.$$

Then

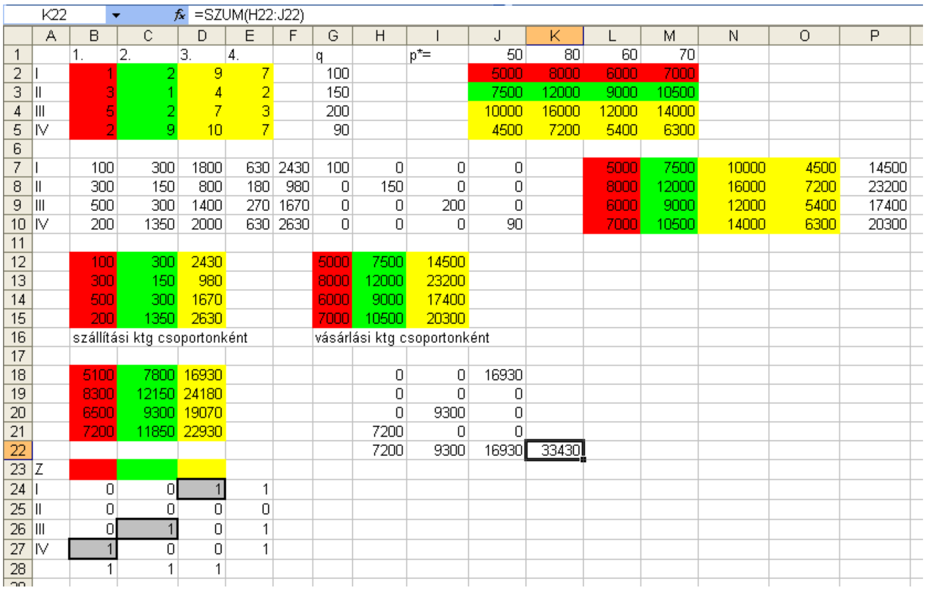
$$\mathbf{F} = \begin{bmatrix} 5100 & 7800 & 16930 \\ 8300 & 12150 & 24180 \\ 6500 & 9300 & 19070 \\ 7200 & 11850 & 22930 \end{bmatrix}.$$

Solution of the task:

Possible place		Cluster
I	→	[3;4]
III	→	[2]
IV	→	[1]

Logistics costs amount up to 33 430.

Excel task and its solution in relation with the model:



K22 =SZUM(H22:J22)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1		1.	2.	3.	4.		q		p*=	50	80	60	70			
2	I	1	2	9	7		100			5000	8000	6000	7000			
3	II	3	1	4	2		150			7500	12000	9000	10500			
4	III	5	2	7	3		200			10000	16000	12000	14000			
5	IV	2	9	10	7		90			4500	7200	5400	6300			
6																
7	I	100	300	1800	630	2430	100	0	0	0		5000	7500	10000	4500	14500
8	II	300	150	800	180	980	0	150	0	0		8000	12000	16000	7200	23200
9	III	500	300	1400	270	1670	0	0	200	0		6000	9000	12000	5400	17400
10	IV	200	1350	2000	630	2630	0	0	0	90		7000	10500	14000	6300	20300
11																
12		100	300	2430			5000	7500	14500							
13		300	150	980			8000	12000	23200							
14		500	300	1670			6000	9000	17400							
15		200	1350	2630			7000	10500	20300							
16		szállítási ktg csoportonként					vásárlási ktg csoportonként									
17																
18		5100	7800	16930				0	0	16930						
19		8300	12150	24180				0	0	0						
20		6500	9300	19070				0	9300	0						
21		7200	11850	22930				7200	0	0						
22								7200	9300	16930	33430					
23	Z															
24	I	0	0	1	1											
25	II	0	0	0	0											
26	III	0	1	0	1											
27	IV	1	0	0	1											
28		1	1	1												

Fig. 6. Worksheet of the task

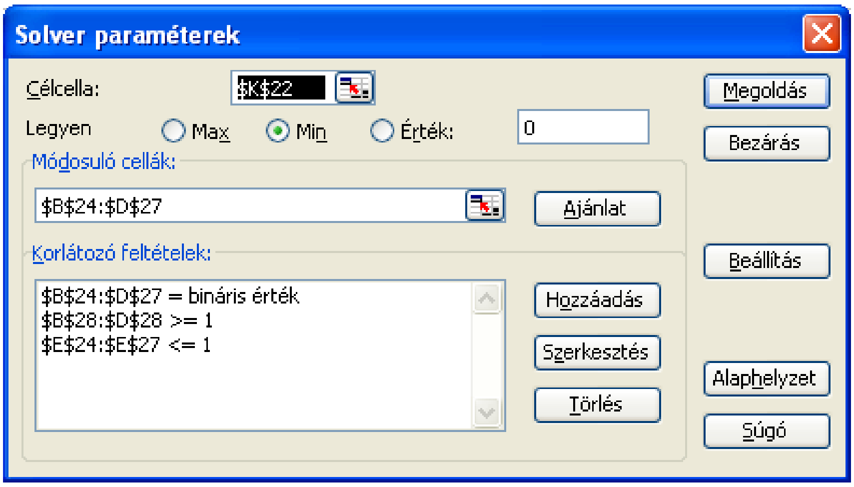


Fig. 7. Setup 1 of Solver

Fig. 8. Setup 2 of Solver

4 Conclusion

Clearly indicate advantages, limitations, and possible applications.

In this paper an attempt has been made for drawing up the mathematical model of site selection related to a novel production method. The model consists of two parts:

- a clustering model, and
- a center assignment.

The models illustrate very well the naturally coherent demand places, i.e. those belonging into the same group, and those best possible places connecting to these clusters.

The models can be handled easily, only the determination of the belonging distances seems to be a bit difficult. However, as it has been illustrates in the above sample task, too, the size of tasks belonging to the models could be very large. Because of this fact, it is reasonable to elaborate such method for the task that will not increase the size that of up to an unmanageable level. The elaboration of such method has been in progress.

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