Objective function analysis of biomass processing plants location

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Abstract

Production of energy carriers by using biomass, among them the biofuels, has come recently into highlight mainly based upon the consequences of surveys that forecasting limited availability of fossil energy sources and, on the other hand, as a result of efforts aiming at decreasing environmental load causing climate change. Despite of its apparent simplicity, the decision making on involving biomass into the distributed energy generation process or any other energy systems that might open up new vistas requires great number of factors to be considered for the sake of economic feasibility. It is easy to understand that among these factors the investment costs, as well as the market prices of fossil energy carriers and electricity, the expenses of distribution and sale, etc. are of focal importance. It is, however, difficult to understand the reason why get logistics aspects still almost negligible weight among the main decision factors. The aim of the current study is to draw the attention to the importance of the topic. The methodology proposed might help decision makers in profound analysis of biomass as a feedstock supply system and in appraisal of efficiency and effectiveness of the entire supply chain. The model can be suitable both for determination of location sites of processing and service plants based on logistics aspects and for planning the biomass supply system.

Key words: biomass, optimization, distance function, location

JEL Classification: C61

1 Introduction

For today, the wording has become generally known according to which the biomass is the "total amount of organic material and living being in a given habitat" [5], however, only a certain proportion that of can be used for energy generation. According to the current level of technology mainly but not exclusively feedstock of vegetable origin (so called *phyto-mass* – and this is what we focus on in what follows) can be used for practical energy purposes. In relation with this feedstock besides the type and variety, the yield, the convertible energy content, etc. also the spatial availability of biomass is an additional limiting factor from the point of view of reasonable and profitable energetic utilization. Satisfying energetic needs according to expectations is possible only on the base of uniformity in time that requires a continuous feedstock supply of processing plants in which the efficient logistics contribution is of basic requirement (Fig. 1).

Analyses show that the biomass base is definitely considerable in Hungary and, this potential could be further increased by targeted cultivation of plants aiming at energy source production as a consequence of which also certain agricultural production problems could be positively affected.

Fig. 1 Theoretical system of energetic utilization of biomass

In general, technological, environmental and availability characteristics of practical utilization of renewable energy sources inhere also limitations; their efficient utilization can not always be targeted. Biomass, however, constitutes the largest base among the renewable sources that, for the most part, could also be increased. Parallel with the increase of demand for the energy (direct heat, or solid, liquid and gaseous fuels, respectively) generated from such sources, the development of supply chains supporting the realization, along with all their elements and full infrastructure should also be targeted with the same rate at least. Based upon logistics view of the process, under supply chain the macro system of utilization could be understood which includes each of stakeholders from the place of origin up to the place of actual utilization. Today, it is not enough to take into consideration what economic advantages are offered by a bio-diesel or a biogas plant, but “the interconnected subsystems of supply and service processes need to be comprehensively integrated by scientific aspects” [6] in the complete system production. For this purpose, the appropriate set of means is offered by logistics which is, among others, able to apply the methods of operation research also in this field by using the hardware based on the development results of micro-electronics.

Besides a number of other factors the cost efficient supply logistics is a prerequisite of the competitive biomass based energy production. It must be, however, taken into consideration, that the increasing demand for utilization of biomass increases also the complexity of the supply system. Today, there are such comprehensive planning methods needed that, besides the traditional cost aspects, take also other factors into account, such as geographical location of production plots and how are these plots accessible (application of harvesting and collection machinery, vehicles, routes, etc.) as well as leave not out of consideration the competition among the users different methods for utilization of available biomass. Under such circumstances the location of pretreatment and processing plants is of basic importance in given cases. On the base of this approach it can be seen a great number of problems are raised by the location of a processing plant (C marking will be used in what follows) for production of bio-fuels or energy carriers the solution of which requires already the application of higher mathematical methods. It is obviously advantageous to locate the processing plant as close to the places of providing raw material or feedstock (marked with S in what follows) as possible. These points mean mainly agricultural production sites and forestry from which places the feedstock is made available as by-products or main crops if the production is targeted for providing energy plants for the processing centre or could also be places of plants where there are agricultural products processed and, as a result, the produced by-products can be well utilized for energetic purposes. The location of such plants is fixed in

most cases. It is also obviously advantageous if plants for utilization or deposition of by-products of energy generation (marked with R in what follows) as well as those points of use or consumption (final destination of energy marked as U points) which constitute the direct link with the customer or consumer in the supply chain. Consequently, the site of location receives an important role. As, from the point of view of location, the material flow takes place between and among these points the definition of the centre will obviously be the function of costs, distance and time of the flow. This is illustrated in Fig. 2.
The problem of the task is that the individual *d* values are differently calculated. As these values integrate all those factors that serve as objective functions regarding the location there could be other factors regarding the inbound transportation, the outbound transportation to the users or to the reutilization, respectively. Moreover, in the first examination of these cases we are not able to define even the effect of individual factors on the objective function. In the following part of the study we try to find a solution for this problem.

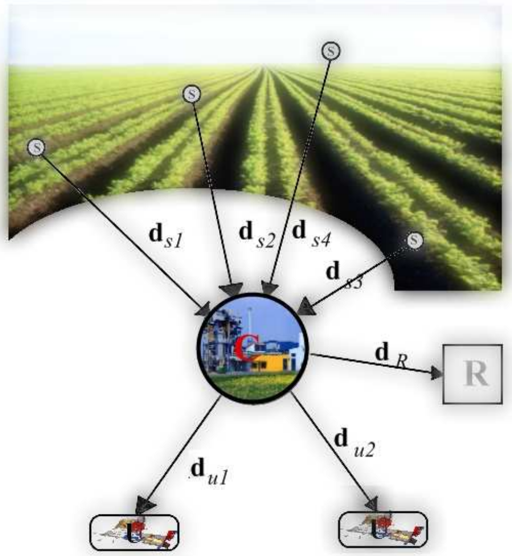

Fig. 2 The structure of the system

Processing in a centre
The simplest however not the best solution is if we set up a central processing plant for the storage and processing of the feedstock. The reason why this solution is considered to be not the most appropriate is that the expenses may significantly increase because of long distance transportation.

Location of more processing plants
In this case the processing plants are located around feedstock centers and, regarding the number and actual place of which the hierarchical clustering methods provide solution. As the most important notion of clustering is the distance there is a need for formation an efficient distance notion which serves also as an objective function at the same time.

3 Definition of the problem

As the first step in definition of the problem that objective function needs to be found for which the optimization of clustering will be accomplished and also the simulations of the later system, that provide the framework of actual operation, will be based upon that. In what follows, the objective function will be considered as *distance function*, although this will not be a real distance describing function.

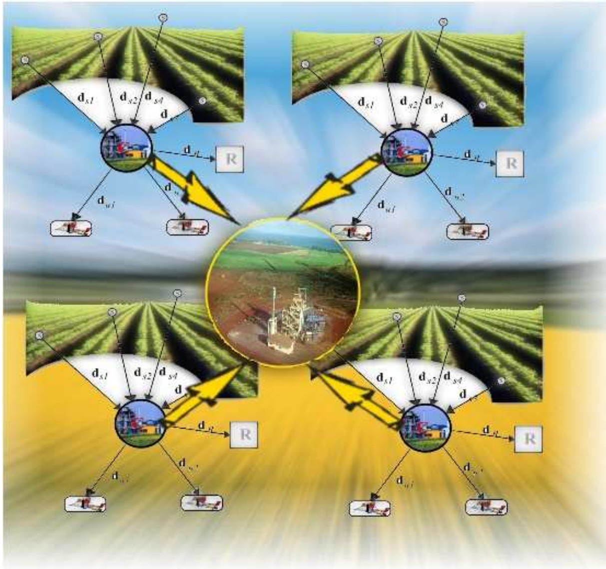

Fig. 3

General characteristics of the distance function

A very complicated problem in our task is the formation of an applicable distance notion. As this character will control the solution of the task great number of factors needs to be taken into consideration. It is obvious, that the virtual distance depends not only on the physical distance, but also on time and cost factors as well as on other influencing factors like the type of vehicle, road quality, etc.

Let us consider the simplest case in which the physical distance to the preliminary defined center is influenced by the specific cost of transportation (it is surely depending on the transported volume), the transportation time and the physical distance (including possibly the road quality, etc.) itself. Besides, take in also the quality parameter which means the aggregate of scaled value of other characters. For the first approach, the following function seems to be simple:

$$\delta(x;P_1;P_2)=\lambda_1 xc(x,P_1;P_2)+\lambda_2 t(x,P_1;P_2)+\lambda_3 d(x,P_1;P_2)+q(x,P_1;P_2), \quad (1)$$

where:

$c(x;P_1;P_2)$: is the specific transportation cost between the $P_1;P_2$ points in case of *x* volume;
$t(x,P_1;P_2)$: is the transportation time function in case of *x* volume;
$q(x,P_1;P_2)$: is a quantitative quality function „lower value in case of better quality” (non-dimensional);
$d(x,P_1;P_2)$: is the transportation route function in case of *x* volume;
λ_1: is the cost involvement factor (with the dimension of 1/monetary unit)
λ_2: is the time involvement factor (with the dimension of 1/time unit)
λ_3: is the distance involvement factor (with the dimension of 1/distance unit)

The function (1) is appropriate in that sense that it is increasing as the cost, time and transportation distance as well as the quality functions are increasing. Moreover, it would be very good from the point of view of problem solving as its components could be individually analyzed making the sensitivity analysis simpler. There are though problems that can not be left out from considerations. It is obvious that the parameters are dependent even on each other. It may happen that the decrease of transportation time would considerably increase the transportation costs and vice versa. Apparently, the transportation time depends on the length of the route; however, it is not true in each case. Let us consider the following simple example.

Between points A and B the shortest path is the cart-road, but it takes longer travel in time than that on the hardly longer highway, however the use of highway is more expensive (because of the toll). For this reason the function can not be used in general. (At such "distance function" does not prevails the triangle inequality. Prevails however the

Fig. 4 Problem with the traditional distance function

$d(A,A)=0$,

$d(A,B) \geq 0$,

but unfortunately $d(A,B) \neq d(B,A)$,

or not sure, at least.)

In case of $\delta(x;P_1;P_2)$ we lay following claims against the function:

1) Let the function be of non-negative.
2) Let the function be of monotonically increasing with the time and cost variables, moreover, partially differentiable.
3) It is recurrently continuous function according to volume.
4) The parameter is monotonically increasing by the volume, i.e. if $x_1 < x_2$ then $t(x_1,P_1;P_2) \leq t(x_2,P_1;P_2)$.

   This latter constraint raises that the time function significantly depends on the transportation quality. This is obvious, as in case of larger quantities road transportation may be substituted by air transport, for instance, consequently the time can be reduced. The 4) improved:

4a) $x_1 < x_2$, then $t(x_1, P_1; P_2, q) \leq t(x_2, P_1; P_2, q)$.

5) With the following conditions the time component is strictly monotonic with the distance:

a) the volume is constant: x_0

b) transportation quality is constant: q_0

c) $d(P_1; P_2) < d(\hat{P}_1; \hat{P}_2)$

then

$t(x_0, q_0, d(P_1; P_2)) < t(x_0, q_0, d(\hat{P}_1; \hat{P}_2))$

6) At constant distance and transportation quality the cost component is of recurrently linear with the volume.

7) The cost function component depends on the distance in case only if the volume and transportation quality; in this case it is monotonically increasing with the distance.

8) The costs may decrease with the improvement of the transportation quality or increase, but it significantly impact on the cost component.

9) In general case there is no relation between the cost and the time component.

In general let us consider the following form of the distance function:

$$\delta(x; P_1; P_2) = \lambda x c(x, d(P_1; P_2;); q(P_1; P_2)) + (1-\lambda) t(x, d(P_1; P_2); q(P_1; P_2)), \qquad (2)$$

where $0 \leq \lambda \leq 1$

The above distance function unambiguously reveals also that there could be only three optimization goals for the solution.

1) Optimization for the cost: $0{,}75 \leq \lambda \leq 1$;
2) Optimization for the time-cost: $0{,}25 < \lambda < 0{,}75$;
3) Optimization for the time: $\leq \lambda \leq 0{,}25$

Special distance functions

The following cases give exacter distance functions in relation with special locations. The classification is made hierarchically mainly because of solution methods.

Time optimized service

As first, let us consider the case in which there is no cost constraint and a certain given volume has to be transported. In this case the distance function consists only of the time component:

$$\delta(x; P_1; P_2) = t(x, d(P_1; P_2); q(P_1; P_2)). \qquad (3)$$

The function (3) derives from the above generalizations. It is recurrently linearly increasing from the volume. Let us examine its relation with the distance:

Let the transportation be between $P_0 \to P_n$ points. Let further be all possible route selection nodes $P_1, P_2, \cdots, P_{n-1}$. The possible paths among these nodes are given by the **D** hyper-matrix. This matrix is obviously a rare one as between many nodes there is no actual link and, in our case, it could only be exceptional if we take also the return path into consideration. Apparently, the matrix elements may have not only one value in order to form an appropriate structure and, of course, its elements need to be vectors. The matrix will be of three-dimensional in which the first dimension are formed by the possible starting points, the second is constituted by the destinations, while the third dimension is characterized by the quality road selection. The form of the graph is a non-roundtrip weighted graph with more edges between tow vertices although this graph may also have the form in which there is only one edge leading from each vertex to the other. The restructuring can be accomplished as depicted in the Fig. 5.

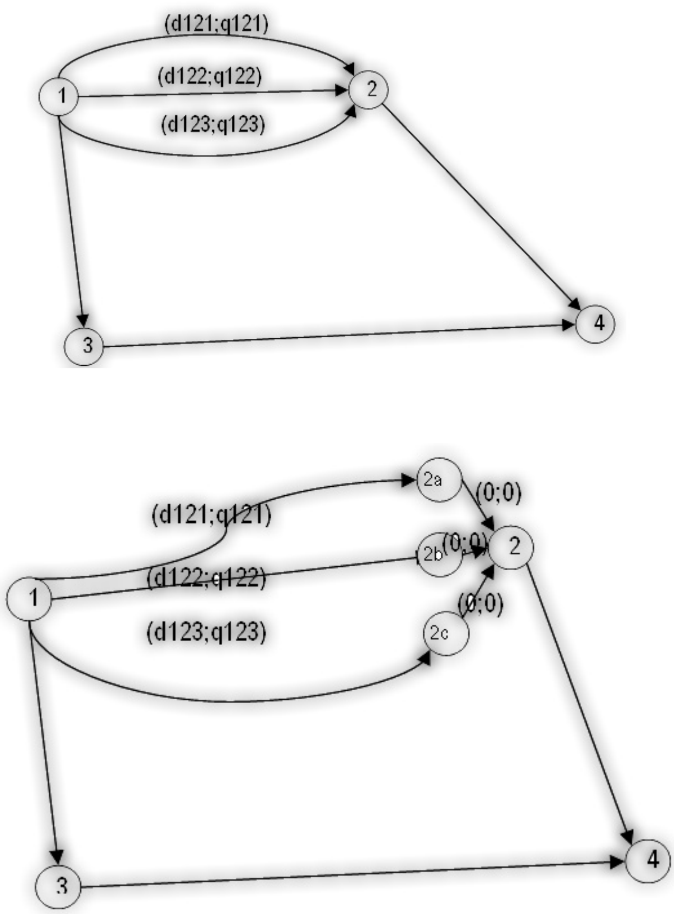

Fig. 5: Rationalization of the directed graph

Let us structure the following hyper-matrix the elements of which are three-dimension vectors:

$$D_{ijk}\left[(t;d;q)\right], \tag{4}$$

Where

i: is the starting vertex;

j: is the destination node;
k: selected possibility;
t: duration for the *k* possibility between P_i, P_j;
d: distance for the *k* possibility between P_i, P_j;
q: quality for the *k* possibility between P_i, P_j;

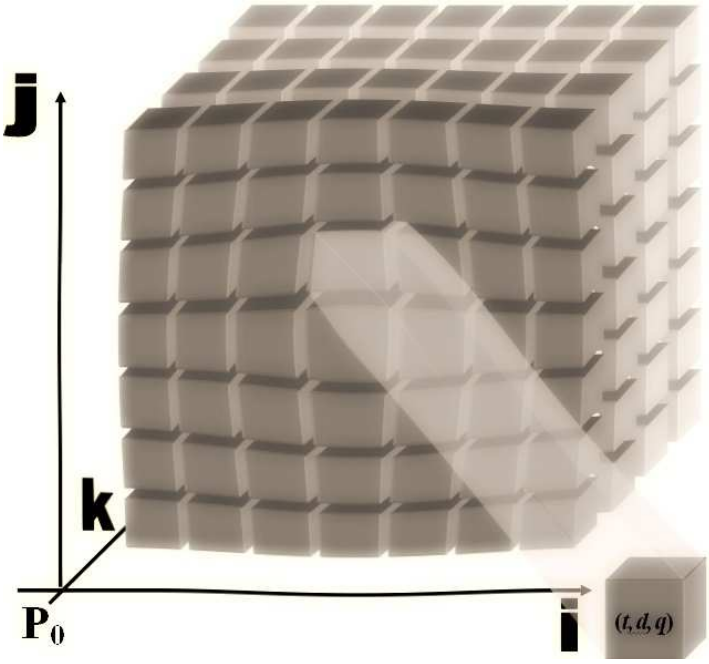

Fig. 6: The hyper-matrix

Task: let us find the shortest path in time between $P_0 \to P_n$ in a manner that no each node has to be touched! If there would not be more possibilities between certain two points the task could easily be solved by the traditional PERT or CPM methods. As there are more possibilities available, the multiple routes are considered as if leading between independent nodes and "between its own" we consider infinite time for the sake of traditional solution. It is even more interesting if there is a change accomplished along also the *k* axis (that is, there is a quality transportation executed). This would mean the change of vehicle and also the road quality changes. In this case there might appear a time factor, as well, in the course of the change. This is included in another matrix.

$$V_{ijk}\left[(t;d;q)\right]. \tag{5}$$

In this case also these values need to be added to the full time, and need to be taken into account in the course of optimization, respectively.

3.4 Cost limited time optimized service

In this case, even further on, the goal of optimization is the time of accomplishment, however, there will be a cost constraint taken into account. For the reason the elements of the above **D** matrix are of four-dimensional vectors. The fourth dimension will be the specific cost, that is, a cost unit calculated for one unit of volume and that of route. In the course of searching for

the route attention should be paid to the fact that along the search for optimal time the predetermined cost constraint must not be exceeded.
It may occur that there is no solution for the task and in such cases the cost constraint needs to be modified or the transportation between two certain point needs to be rejected.

Time optimized service

This case is not significantly different from the previous survey. There is no time limit, and transportation of one concrete quantity has to be accomplished. In this case the distance function consists of distance component:

$$\delta(x;P_1;P_2) = xc(x,d(P_1;P_2;);q(P_1;P_2)), \tag{6}$$

We repeatedly create our hyper-matrix with the difference that the first component of the element vector will be the specific cost. The solution is fully similar.

Time limited cost optimized service

The solution will be the same as in the case limited time function.

Distance function service

It follows from the foregoing that the analysis of the general case is no longer difficult. We repeatedly create our matrix with four-dimensional vectors in which the two components are simultaneously and at the same manner present, namely the time and the cost. In the course of searching for route first the cost and time values are summarized then the distance function values will be created to the found route with preliminary fixed λ factor. Following this, that route will be selected from those being possible for which this value will be the least. Here, the method differs from that of before. The reason of this deviation is that the partial minimums will not surely give the final minimum. Unfortunately, in this case the final minimum can only be found if we survey each possible case. In case of many vertices this could be a very large and long lasting task.

4 Creation of the general objective function, optimizing goals

With the application of the method outlined above there could be concrete values calculated to the distance function at given distances and qualities by typical patterns selected from a given region, the country or a larger area. When selected these should possibly connect to location nodes. The above theoretical approaches demonstrate for the practice that the general objective function is determined by the distance covered, the transportation time, the quality of the road and transportation, the transported volume, as well as the specific transportation cost. In order to obtain a tool for practical definition of the location objective function the relation between the above parameters and the final cost should be found. For this purpose the best solution is offered by the artificial neural networks. The connection function will be denoted by F in what follows. As soon as we have connected the input parameters with the output parameter, we are able to generate such "cost" values on the base of known inputs which can be assigned to the given transportation and will produce the nearly exact value. This relation, considering that it is function, is already suitable for optimization. The goal in each case is the determination of the least cost values in relation with the given transportation case. Of course, this is not so simple as certain constraints need to be taken into consideration. Among these the maximum transportation time, the type of vehicle, as well as the connecting parameters like the specific transportation time and the quality of the transportation. For this reason, in special cases the task could be simplified by preliminary fixing of certain parameters.

The type of transportation vehicle and the specific transportation cost alter between narrow limits

For the sake of flexibility, in this case the only constraint is that the specific transportation cost alters between fixed narrow limits depending on the type of the vehicle. The known specific transportation cost depends closely hyperbolically from the transported volume, that is:

$$c^T(q) = \frac{C_0\left(int\left(\frac{q}{Q}\right)+1\right)+qc}{q} = \frac{C_0\left(int\left(\frac{q}{Q}\right)+1\right)}{q}+c\,, \tag{7}$$

.

where

C_0 : is the base cost related to a transportation vehicle;

c : specific relative transportation cost related to a product unit;

q : transported volume;

Q : capacity of the transportation vehicle.

The characteristic curve of the function is depicted in Fig. 7.

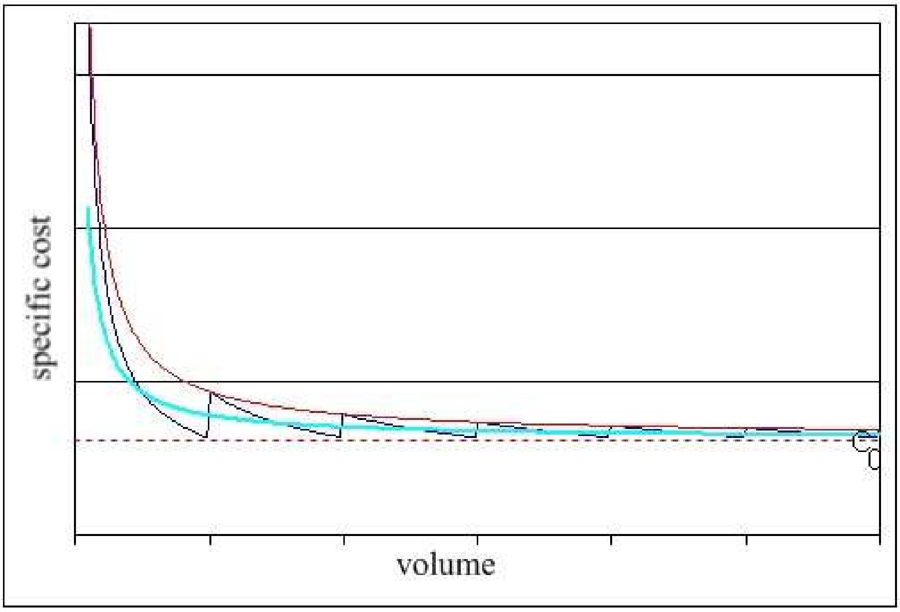

Fig. 7 The specific transportation cost as a function of the volume

Enveloping curve from above:

$$c^T(q) = \left(\frac{C_0}{Q}+c\right)+\frac{C_0}{q} = B+\frac{C_0}{q}\,, \tag{8}$$

where B is a constant depending only on the vehicle.

In the Fig. 7 there is another curve, as well, namely, we are searching for the hyperbolic function which best adjusts to the original function, where q is considerably larger than Q.

In the course of approximation we survey the difference between the area of the two functions and are looking for the parameter *a* for which the result is zero or close to that.

$$\int_{q_{min}}^{q_{max}} \left\{ \frac{C_0\left(int\left(\frac{q}{Q}\right)+1\right)}{q} + c - \left(\frac{C_0}{Q} + c + \frac{a}{q}\right) \right\} dq = \\ = \int_{q_{min}}^{q_{max}} \left\{ \frac{C_0\left(int\left(\frac{q}{Q}\right)+1\right)}{q} - \frac{a}{q} - \frac{C_0}{Q} \right\} dq \,. \tag{9}$$

Let us decompose the minimum and maximum transportation interval for equidistant part intervals according to the Q volume.

$$q_{min} = q_0 ; q_1 ; \cdots ; q_n = q_{max}$$

$$T = -\int_{q_{min}}^{q_{max}} \left\{ \frac{a}{q} + \frac{C_0}{Q} \right\} dq + \int_{q_{min}}^{q_{max}} \frac{C_0\left(int\left(\frac{q}{Q}\right)+1\right)}{q} dq = \\ = -\frac{C_0}{Q}\left(q_{max} - q_{min}\right) - a\,ln\frac{q_{max}}{q_{min}} + \int_{q_{min}}^{q_1} \frac{C_0}{q} dq + \sum_{i=2}^{n} \int_{q_{i-1}}^{q_i} \frac{C_0 i}{q} dx. \tag{10}$$

Completing the (10) integration operation:

$$T = -\frac{C_0}{Q}\left(q_{max} - q_{min}\right) - a\,ln\frac{q_{max}}{q_{min}} + \\ + C_0\,ln\,q_1 - C_0\,ln\,q_{min} + 2C_0\,ln\,q_2 - 2C_0\,ln\,q_1 - \cdots nC_0\,ln\,q_n - nC_0\,ln\,q_{n-1}. \tag{11}$$

then simplified further:

$$
\begin{aligned}
T &= -\frac{C_0}{Q}(q_{max}-q_{min}) - a\,ln\frac{q_{max}}{q_{min}} + C_0(n\,ln\,q_n - ln\,q_{min} - ln\,q_1 - \cdots - ln\,q_{n-1}) = \\
&= -\frac{C_0}{Q}(q_{max}-q_{min}) - a\,ln\frac{q_{max}}{q_{min}} + C_0\left(ln\frac{q_n}{q_{min}}\right) + C_0\left(ln\frac{q_n}{q_1}\right) + \cdots + C_0\left(ln\frac{q_n}{q_{n-1}}\right) = \\
&= -\frac{C_0}{Q}(q_{max}-q_{min}) - a\,ln\frac{q_{max}}{q_{min}} + C_0\,ln\frac{q_n}{q_{min}} + C_0\sum_{i=1}^{n-1} ln\frac{q_n}{q_1} = \\
&= -\frac{C_0}{Q}(q_{max}-q_{min}) - a\,ln\frac{q_{max}}{q_{min}} + C_0\,ln\frac{q_{max}}{q_{min}} + C_0\sum_{i=1}^{n-1} ln\frac{n}{i} = \qquad (12) \\
&= -\frac{C_0}{Q}(q_{max}-q_{min}) - a\,ln\frac{q_{max}}{q_{min}} + C_0\,ln\frac{q_{max}}{q_{min}} + C_0\,ln\frac{n^{n-1}}{(n-1)!} = \\
&= C_0\,ln\frac{q_{max}}{q_{min}} + C_0\,ln\frac{n^{n-1}}{(n-1)!} - \frac{C_0}{Q}(q_{max}-q_{min}) - a\,ln\frac{q_{max}}{q_{min}} = \\
&= (C_0 - a)ln\frac{q_{max}}{q_{min}} + C_0\,ln\frac{n^{n-1}}{(n-1)!} - \frac{C_0}{Q}(q_{max}-q_{min})
\end{aligned}
$$

The zero of function T obtained as a result of the above procedure needs to be found which no longer so complicated task is; the resolution can be reached by numeric methods. With the obtained parameter *a* the

$$T_a(q) = \left(\frac{C_0}{Q} + c\right) + \frac{a}{q}$$

hyperbolic function will be the approximating function which can be well applied for resolutions in case of $q >> Q$ condition.

In the current special case two constants may alter between preset value limits. Practical experiences show that we do not make severe mistake if we characterize the two parameters with the same interpolating parameter (this would mean that the transportation vehicles do not greatly differ from each other).

$$c = c_{min} + \hat{\lambda}(c_{max} - c_{min}),$$

illetve

$$C_0 = C_{min} + \hat{\lambda}(C_{max} - C_{min}) \qquad (13)$$

where

$$\hat{\lambda} \in [0;1].$$

Accordingly the (7)

$$c^T(q;\hat{\lambda}) = \frac{\left[C_{min} + \hat{\lambda}(C_{max} - C_{min})\right]\left(int\left(\frac{q}{Q}\right) + 1\right) + q\left[c_{min} + \hat{\lambda}(c_{max} - c_{min})\right]}{q}. \qquad (14)$$

The transportation cost is formed according to the vehicle and the volume:

$$C^T(q;\hat{\lambda}) = \left[C_{min} + \hat{\lambda}(C_{max} - C_{min})\right]\left(int\left(\frac{q}{Q}\right)+1\right) + q\left[c_{min} + \hat{\lambda}(c_{max} - c_{min})\right] = \\ = qc^T(q;\hat{\lambda}) \qquad (15)$$

The total transportation cost:

$$C^T(q) = C_0\left(int\left(\frac{q}{Q}\right)+1\right) + qc\,,$$

respectively with the approximating function: (16)

$$C_a^T(q) = \left(\frac{C_0}{Q} + c\right)q + a\,.$$

Fig. 8 Transportation cost plotted against the volume

Transportation time depends linearly on the distance and recurrently linearly on the volume

The transportation time depends mainly on the distance, the type of vehicle, the quality of transportation and the volume.

Let

q_0 : the transported volume,

$\tilde{q}$: the quality of transportation (fixed),

$C^T(q_0)$: cost dependent on the vehicle (fixed).

For the calculation of the transportation time let us examine the volume and vehicle capacity ratio first:

$$n = \frac{q_0}{Q_0}. \tag{17}$$

n_0 should denote the number of vehicles being simultaneously available. In this case the number of simultaneous transportations:

$$m = \left\lceil \frac{n + n_0 - 1}{n_0} \right\rceil. \tag{18}$$

Let us consider the loading time linear with the number of vehicles:

$$T_l(n) = T_{l0} + nt_l. \tag{19}$$

Where T_{l0} is a time factor dependent on the type (capacity, etc.) of the vehicle, and t_l is a constant dependent on overlapping. The same relates to the unloading, that is:

$$T_u(n) = T_{u0} + nt_u. \tag{20}$$

The transportation time in the function of the distance:

$$T_t(d, q_0) = dT_t + nt_t. \tag{21}$$

In the above equation T_t is the specific transportation time related to the unit length of path and t_t is a slip because of overlapping.
The time for the return route:

$$T_u(d, q_0) = dT_u + nt_u. \tag{22}$$

Following this, we can already define the transportation time for the total q_0 volume for the case of continuous transportation:

$$T(d, q_0) = (m+1)(T_{l0} + T_{u0} + d(T_t + T)_u) + mn_0(t_l + t_u + t_t) + \left\{\frac{n}{n_0}\right\}(t_l + t_u + t_t). \tag{23}$$

In the case the time depends not only on the length of path but also on the transportation quality then let us considers the transportation quality matrix $\mathbf{Q} = [q_{ij}]$ $i = 1, \cdots, l$ related to the possible transportation routes and vehicles instead of the *d* parameter. (Remark: there is smaller value belonging to higher quality.) In this matrix d_i is the length of *i*-th path. Moreover, let us assume that the transportation parameter of higher value means a faster transportation. The quality value is scalar without dimension. In such case the transportation by *i* type vehicle on a *j* type road:

$$T_{ij}(q_0)=(m_i+1)\left(T_{l_0ij}+T_{u_0ij}+d_j q_{ij}(T_{ti}+T_{ui})\right)+m_i n_{0i}(t_{li}+t_{ui}+t_{ti})+\left\{\frac{n}{n_{0i}}\right\}(t_{li}+t_{ui}+t_{ti}). \quad (24)$$

Consideration of time constraint

In case the transportation time has an upper limit it can be modified by appropriate selection of the transportation vehicle and route. It is obvious, that in this solution the cost should be kept at the lowest value.

A truckload volume

In this simplified case we assume that there is at least one vehicle with a capacity of transporting the total volume.

Let the vehicles meet the following requirements:

The specific relative transportation cost is a matrix related to the given route: $C=[c_{ij}]$ $i=1,\cdots,l$; $j=1,\cdots,k$, where i denotes the possible types of vehicles and j the possible routes. In the case of a path on which the transportation is not possible with the given type of vehicle, the matrix bears the ∞ symbol. The different possible transportation distance vector: $\underline{d}=[d_j]$, $j=1,\cdots,k$. Moreover, let us consider that the transportation quality for the possible routes and vehicles is described by the $\mathbf{Q}=[q_{ij}]$ $i=1,\cdots,l$ matrix. (Remark: there is smaller value belonging to higher quality.) Let us assume that the transportation of better quality means a faster transportation. However the relation with the cost is already not so simple. This is the reason why we will apply a $\mathbf{C}^q=[c_{ij}^q]$ supplementary specific cost factor between the vehicle and the route instead of the quality matrix. As a result the transportation cost (without loading and unloading) depending on type of vehicle and the length of path:

$$\mathcal{C}_{ij}=q_0\left(c_{ij}+c_{ij}^q\right)d_j+C_{0_i}. \quad (25)$$

In case we consider also the expenses of loading and unloading depending on the volume:

$$\mathcal{C}_{ij}=q_0\left[\left(c_{ij}+c_{ij}^q\right)d_j+c_{il}+c_{iu}\right]+C_{0_i}. \quad (25)$$

The $c_{il};c_{iu}$ denote specific loading and unloading costs.

As according to the assumption of simplification in which the total volume can be transported by one vehicle the (24) will appear in the following form:

$$T_{ij}(q_0)=T_{l_0ij}+T_{u_0ij}+d_j q_{ij}(T_{ti}+T_{ui}). \quad (26)$$

Let T^C denote the time constraint:

$$T_{ij}(q_0)=T_{l_0ij}+T_{u_0ij}+d_j q_{ij}(T_{ti}+T_{ui})\le T^c, \quad (27)$$
$$\mathcal{C}_{ij}\to min!$$

4.3.2 Volume for multiple truckloads in case of homogenous fleet

In this case we assume that the transportation can be accomplished by more vehicles, however the fleet is homogenous.

The transportation time is described by the (24). The cost forms as shown below:

$$\mathcal{C}_{ij}(q_0) = q_0\left[\left(c_{ij} + c_{ij}^q\right)d_j + c_{il} + c_{iu}\right] + (m+1)C_{0_i}\,. \tag{28}$$

The model will appear in the following form:

$$T_{ij}(q_0) = (m_i+1)\left(T_{l_0ij} + T_{u_0ij} + d_j q_{ij}\left(T_{ti} + T_{ui}\right)\right) + m_i n_{0i}\left(t_{li} + t_{ui} + t_{ti}\right) + \left\{\frac{n}{n_{0i}}\right\}\left(t_{li} + t_{ui} + t_{ti}\right) \le T^c \tag{29}$$
$$\mathcal{C}_{ij}(q_0) \to min!$$

Volume for multiple truckloads in case of inhomogeneous fleet

This is the most complicated case as there must be more factors taken into consideration. First of all, those possible transportations need to be selected in which the applicable vehicle types may be different. From among these possibilities those must be selected which meet the time constraint assumption. Finally, from among the possibilities remained that case must be selected which is of the least cost. As it can be seen also from the previous cases, this is a rather complicated task as the transportations are overlapping continuous ones as opposed to those of integrated. For this reason there may occur even queuing at loading and unloading.

First step: The possible transportations

Let us assume that the volume has not changed i.e. q_0. Moreover, the number of possible vehicles is *K*. The capacity of vehicles is described by the $\mathbf{Q} = [Q_i]$ vector, where $i = 1,\cdots,K$. In this case each of transportations is appropriate for which

$$q_0 \le \sum_{i=1}^{K} n_i Q_i < q_0 + \max_i Q_i\ ;\ i = 1;\cdots;K \tag{30}$$

Here, the maximum assumptions seems to be too strong, but taking the case into account when each vehicle of smaller capacity in on the way, maybe it is more beneficial to set on the way one free vehicle of larger capacity than to wait for the one of smaller capacity being on the way. Further on, let us assume

$$\mathrm{S} = \begin{bmatrix} n_{11} & \cdots & n_{1K} \\ \vdots & & \vdots \\ n_{\overline{K}1} & \cdots & n_{\overline{K}K} \end{bmatrix};\ n_{ij} \in \mathbf{N}\,. \tag{31}$$

where $\overline{K}$ denotes the number of all possible transportation cases that meet the requirement of (30).

Second step: Transportations that meet the requirements of time constraint

In this step we exclude the possibilities from above $\overline{K}$ set which do not meet the time constraint. Here, we use the results of the previous 4.3.2 section. Let us consider the $\underline{S}_r = [n_{r1};\cdots;n_{rK}]$ vector $\left(r \in \{1;\cdots;\overline{K}\}\right)$. We further simplify the conditions in a manner that also the truck transporting not a full load will be considered as ones with full load. (In case of transportation of large quantities it does not cause any problem, because the related excess time and cost is negligible as compared to the total. In case of transportation of smaller quantities the task can be easily solved manually, based on practical experiences.)
Following this, the (24) transportation time will be applied for the different types of vehicles:

$$T_{rij}(n_{ri}Q_i) = n_{ri}\left(T_{l_0ij} + T_{u_0ij} + d_j q_{ij}(T_{ti} + T_{ui})\right) + n_{ri}Q_i(t_{li} + t_{ui} + t_{ti}). \tag{32}$$

Assumed that the vehicles do not obstruct each other in the course of transportation the transportation times can be optimized for each vehicle. (Otherwise, a slowing factor that is dependent on the volume has to be installed.)
Let

$$T_{ri}^{min} = \min_j \{T_{rij}(n_i Q_i)\} . \tag{33}$$

as well as in relation with the cost:

$$\mathcal{C}_{rij}(n_{ri}Q_i) = n_{ri}Q_i\left[\left(c_{ij} + c_{ij}^q\right)d_j + c_{il} + c_{iu}\right] + n_{ri}C_{i0_i} . \tag{34}$$

Let us select all possible varieties of transportation and if the (35) is performed then this transportation performs the time constraint:

$$\max_i T_{ri}^{min} \le T^c . \tag{35}$$

In case the transportation does not meet time constraint then this one is excluded from the following examinations. If the time constraint has been met each of transportations related to individual vehicles could be deemed as appropriate for which

$$T_{rij}(n_i Q_i) \le T^c . \tag{36}$$

Let us compose all possible variation of these. Let the number of these be *M*, and r_m one of these variations. In this case the transportation cost for this variation:

$$C_{rr_m} = \sum_{i=1}^{K} \mathcal{C}_{rij}(n_{ri}Q_i). \tag{37}$$

The minimum of these values will be the optimum cost of the given variation.

$$C_r = \min_{r_m} C_{rr_m} . \tag{38}$$

Third step: Transportations that meet the requirements of cost constraint

Finally, from among the possible transportations we select those for which the cost is the least:

$$C = \min_{r} C_r . \tag{39}$$

Apparently, the above cases seem to be ideal many times, however there could be queuing at loading and unloading in case of overlapping and, as mentioned, the vehicles may obstruct each other in case of transportation of large quantities. These cases can easily be derived from the above equations if further adjusting time and cost parameters are considered.

5 Conclusion

The possible agricultural supply centers need to be found in a manner that the feedstock density should be largest in their vicinity. In this paper we examined the distance function that can be applied for locations. The analysis of the distance was accomplished in general. We examined how is it possible to calculate the cost while we take also the quality of transportation and the time factor into account.

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